An Application of the Definite Integral - Cup Activity

In this activity you will use calculus techniques to solve a problem involving Volume and Surface Area. Your calculus will then be verified by using formulas from geometry.

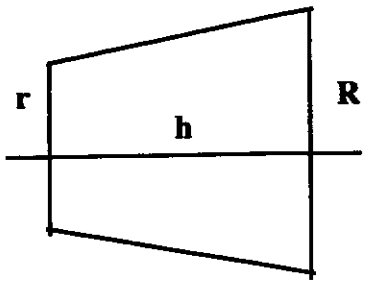
I. Introduction:

A cup can be viewed as a "solid of revolution" by rotating a straight line segment about the x-axis as shown to the right. The only measurements we need to find are:

Radius of the top of the cup (R): _____

Radius of the bottom of the cup (r): _____

Height of the cup (h): _____



In order to find both the volume and surface area of the cup we need to know the **equation of the line** formed by the **edge** of the cup. We can find it because we know two points on the line! The two points are:

(_____ , _____) and (_____ , _____).

Use these points and find the equation of the line. Show your work.

The equation of the line that represents the edge of the cup is: $y =$ _____

II. Volume:

1. Using calculus, find the **volume** of the cup (in cm^3). Write the integral that you are using and evaluate it using the **Fundamental Theorem of Calculus**. (You need to do a little algebra before finding the antiderivative.) Show all of your work.

Volume = _____ (You can check your answer using **fnInt** on your calculator.)

2. The volume of the cup can also be found using the geometry formula for the **volume of a cone** ($V = (1/3)\pi r^2 h$). To make a cone out of the figure above, extend the line segment of the edge of the cone so it crosses the x-axis. We need to know the **x-intercept**. Find it!

x-intercept: (_____ , _____)

Now, the volume of the cup is the volume of the ***big cone*** minus the volume of the ***top cone***. Using the geometry formula above, write the expression for the volumes of the respective cones and evaluate it.

Volume = _____ - _____ = _____

Is this close to your calculus answer? How close? _____

III. Surface Area:

1. The **surface area** of a solid of revolution can be found by the complicated calculus formula:

$$SA = 2\pi \int_a^b f(x)\sqrt{1 + (f'(x))^2}\,dx$$ (where f(x) is the equation for the edge of the cup.)

Using calculus, find the surface area of the cup in cm^2. Show the integral that you are using and evaluate it **with your calculator**. (Not by hand!)

Surface Area = _____ = _____

2. The surface area of the cup can also be found using the geometry formula for the **surface area of a cone** ($SA = \pi rL$ where L is the **slant height** of the cone). We need to know the **two** slant heights to use the surface area formula .
Find them! (Hint: The slant height is the **distance** between two points.)

Slant height of the ***big cone*** = _____

Slant height of the ***top cone*** = _____

Now, the surface area of the cup is the surface area of the ***big cone*** minus the surface area of the ***top cone***. Using the geometry formula above, write the two expression for the surface areas of the respective cones and evaluate it.

Surface Area = _____ - _____ = _____

Is this close to your calculus answer? How close? _____

IV. A Bonus Question:

If the "taper" of the sides of the cup remains the same, how **tall** (the height) should your cup be to be a "Big Gulp" and have a volume of 48 fluid ounces (which is approx 1420 cm^3)? Write an equation to solve this problem, and solve it graphically on your calculator!